PARABOLIC GEOMETRIES FOR PEOPLE THAT LIKE PICTURES

LECTURE 2 WARM-UP: A TALL BUT NARROW WALL

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We, a pedestrian on the Euclidean plane, have encountered an obstacle: in front of us is a bizarre wall, insurmountably tall but narrow enough that it needn't obstruct us if we know how to move on the Euclidean plane.

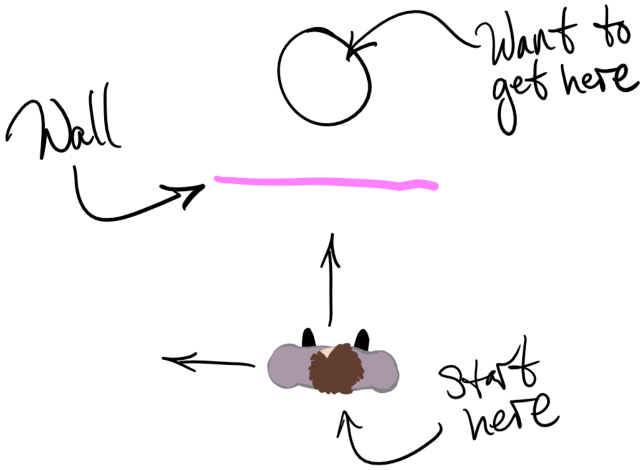

FIGURE 1. Pictorial depiction of the tall but narrow wall "puzzle"

Exercise. *Specifying velocities using the Maurer-Cartan form $\omega_{\mathrm{I}(2)}$, describe a few ways we could get to the other side of the wall.*

For example, we could move leftward until the wall is no longer in front of us, move forward until we are past the wall, and then move rightward so that the wall is directly behind us.

In terms of the basis of $\mathbb{R}^2 < \mathfrak{i}(2)$ where e_1 is unit forward and e_2 is unit leftward, this would be moving with velocity $\omega_{\mathrm{I}(2)}^{-1}(e_2)$ for some time t_1, then with velocity $\omega_{\mathrm{I}(2)}^{-1}(e_1)$ for some time t_2, and finally with velocity $\omega_{\mathrm{I}(2)}^{-1}(-e_2)$ for time t_1. In particular, if g is our initial starting position, then under this sequence of motions, we end up at $g\exp(t_1e_2)\exp(t_2e_1)\exp(-t_1e_2)$.

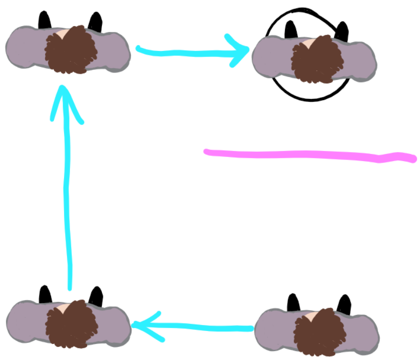

FIGURE 2. Sidestepping to the left, walking forward, then sidestepping to the right

For added entertainment, try to do it with only *one* choice of velocity (you'll probably want to use conjugation to specify it). Alternatively, try to do it via the most obnoxious sequence of motions possible. The only point of this exercise to get used to the Maurer-Cartan form, so just have fun with it.